Unit 1

Textbook: Appendix A, Ch. 1-4

Basics:
- Electrical Quantities (Definitions, Units, Unit Derivations)
- Passive Sign Convention
- Power Delivery
- The Basic Circuit
- Kirchoff's Laws

Equivalent Circuits:
- Fundamental Combinations
- Equivalent Resistances
- Voltage / Current Dividers
- Source Transformations
- Thevenin & Norton

Analysis Methods:
- Node Voltage
- Mesh Current
- Superposition

Unit 1, Section 1: Basics!

Base Assumptions and Electrical Quantities:

There are some basic units and definitions that are vital to everything else, if only so that we can get a bit of intuition as to whats happening and what's being described when hearing a word.

The biggest assumption we make in this class is that all components are lumped and ideal

Lumped - There is no "unit length", everything happens as if it were a point. There's no resistance over a length, it's just resistance

Ideal - Everything behaves theoretically, there's no difference between calculated and actual values

Base SI Units!

(Not much to add, just learn table, will go into detail on a few)

Quantity	Basic Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	degree kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

Current! The net change in charge through a 2-D surface over time

Measured in Ampere's (A) = $\frac{\text{coulomb}}{\text{sec}}$ (C/s) = $\frac{dq}{dt}$

Charge: The amount of force something experiences when placed in an electric field

Measured in Coulombs (C) = $6.24 \times 10^{18} \bar{e}$

$1\bar{e} = 1.6 \times 10^{-19}$ C

We assume that charge is in discreet quantities of the electron, it can be positive or negative, and cannot be created or destroyed

Energy - The capacity to do... something (Work)

Measured in Joules (J) : Nm = Force · Distance

By Conservation of energy, it cannot be created or destroyed

Voltage - The electric potential energy per unit charge (Electric Potential)

Measured in Volts (V) = J/C

Power - The actual doing of work and using of energy per unit time

Measured in Watts (W) = J/s

Most common use is $V \cdot I$: $\left(\frac{J}{C}\right)\left(\frac{C}{s}\right) = J/s$

Resistance - The measure of opposistion to current flow

Measured in Ohms (Ω) = V/A

Inverse is Conductance, measured in Siemens (S) = A/V

Creates a strict ratio known as ohms law that most things in this class, follow

Frequency - How often, of "frequent" something happens
Measured in Hz = 1/s (cycles per second)
Measured in rad/sec (angular frequency)
Often use angular frequency b/c we deal w/ sinusoidal waves in AC
$\omega = 2\pi f$

Derived SI units:

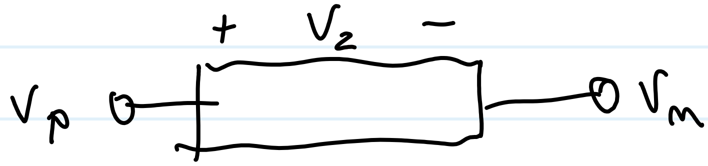
Quantity	Unit Name (Symbol)	Formula
Frequency	hertz (Hz)	s^{-1}
Force	newton (N)	$kg \cdot m/s^2$
Energy or work	joule (J)	$N \cdot m$
Power	watt (W)	J/s
Electric charge	coulomb (C)	$A \cdot s$
Electric potential	volt (V)	J/C
Electric resistance	ohm (Ω)	V/A
Electric conductance	siemens (S)	A/V
Electric capacitance	farad (F)	C/V
Magnetic flux	weber (Wb)	$V \cdot s$
Inductance	henry (H)	Wb/A

Passive Sign Convention (PSC):

A vital concept for this course is the passive sign convention, as it sets consistency and determines the signs of our calculations for everything. It is independent of components, and you will cry if you get it backwards, so don't. It will cause pain.

Key Rule: A positive current direction is one that heads in the direction of a voltage drop

So a drop is positive, but what does that mean? Lets start with just a plain component



Those little plus and minus signs on it are describing the potential relative to the component. The plus sign is the side of the device w/ a higher potential, and the minus sign is the side with the lower potential, if we're following the PSC

In our diagram, following the convention, you might have 3.3V, 5V, 9V, 12V or any other voltage on the positive side, and then 0.5, 1V, 2V, or 0V on the other, the actual values really don't matter, it's just that the voltage of the component is $V_2 = V_p - V_m$, from this formula can you see why a voltage drop is positive for PSC? If it wasn't, we'd say that voltage rises were positive, which would leave a whole lot of negative signs since almost nothing (other than sources) produce a voltage rise.

So.., we know when a voltage is postive under the PSC, and thats when higher potential is on the + side. But calculations almost never happen with just voltage, so we need to think about current too.

Getting the sign right for current isn't terrible, but you have to think about what current is. If you imagine electric current like a river, where water is flowing, then this gets a bit easier.

Water, has some amount of potential energy pretty much all the time. If you put it at the top of a ramp, it will naturally flow down, almost as if we could assign a positive sign to that drop in potential to say "Yay! you obeyed the laws of physics! Good Job!". Do you see where I'm building to?

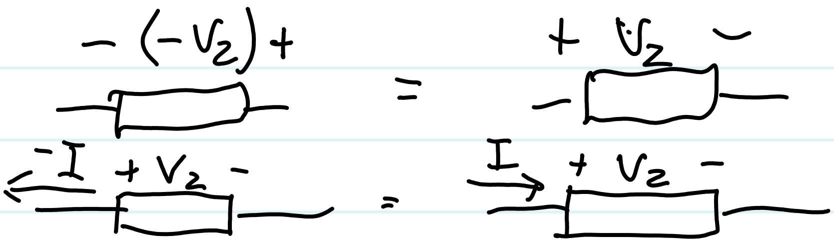
Electric current is the same way, if it's positive it's flowing from a high to a low potential, just like how water moves. If it decides it wants to flow from low to high, something funny is going on so we give it negative sign, because it would be pretty weird if water just started flowing uphill.

So in summary!

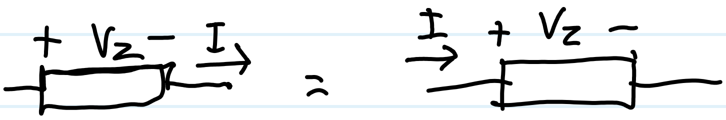
Voltage is positive if the higher potential is on the positive side

Current is positive if it flows towards a voltage drop (the positive terminal of a component)

Quick note:



Don't be tricked by a negative. Negatives mean reversed direction/Polarity



Also... you can shift your current symbol as long as you keep direction the same

Power Delivery!

Armed w/ our new knowledge on the passive sign convention, we can now talk a fairly simple concept, as long as you get your signs right.

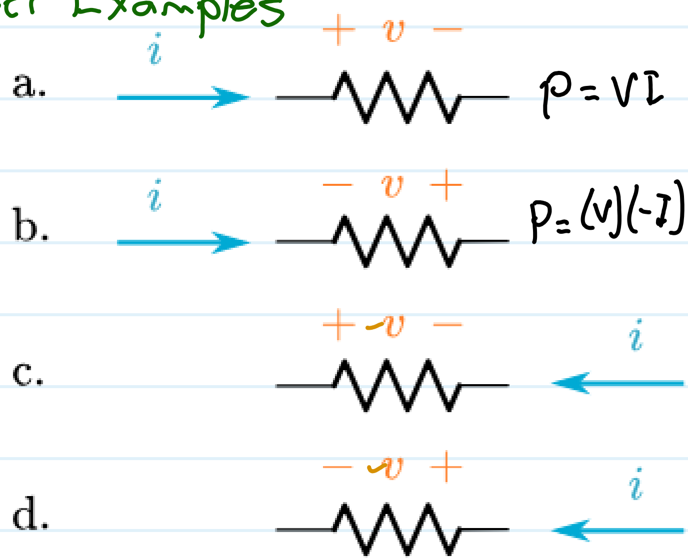
Power is the amount of work done per unit time. If voltage is the work done per amount of charge and current is the amount of charge visiting an area per second, can you think of a way to combine them to get power, which is the amount of work done per second. It might be easier to see from their unit defintions!

$$V = J/C, \quad A = C/s, \quad P = J/s$$

$$\text{Power: } P = VI = \left(\frac{J}{C}\right)\left(\frac{C}{s}\right) = \frac{J}{s}$$

Its important to note that power follows the PSC. Remember, a voltage drop is positive, that means energy being "removed" from the circuit is given a positive sign, and the same follows for power, dissipation (removal) is positive, and delivery (addition) is negative. On the next page are a few examples of how power works

Power Examples



a. $P = VI$ — $(+)(+) = (+)$.
Positive Power = Dissipation

b. $P = (V)(-I)$ — $(+)(-) = (-)$
Negative Power = Delivery

c. $P = (-V)(-I)$; $(-)(-) = (+)$
Positive Power = Dissipation

d. $P = (-V)(I)$; $(-)(+) = (-)$
Negative Power = Delivery

Do note, all of this was discussed before ever showing a single "real" component, because they apply to everything. Keep this in mind later on.

The Basic Circuit:

Considering this course is called circuit analysis, we're gonna deal with a lot of circuits, but lucky for us, we're (generally) going to be dealing with a special kind of circuit called "lumped, linear, time-invariant" which are some special terms that describe everything but the source

Lumped - The components act like a "point", where once you pass it, the properties of the circuit are immediately changed

Linear - For every single component in the cuircuit, voltage is directly porportional to current ($V = \alpha I$)

This looks like a formula we're about to learn about...

Time-Invariant - The properties of the circuit do not vary w/ time. This isn't true once we hit AC power.

Our circuits will also have 2 types of components: passive and active

Passive - Components that do not generate power. They only get done to

Active - Components that deliver power to the circuit

Sources:

There are a total of 4 types of sources, but the relations between them can be thought of in 2 ways:

- Sources
  - Independent
    - Voltage
    - Current
  - Dependent
    - Voltage
    - Current

- Sources
  - Voltage
    - Indep.
    - Dep.
  - Current
    - Indep.
    - Dep.

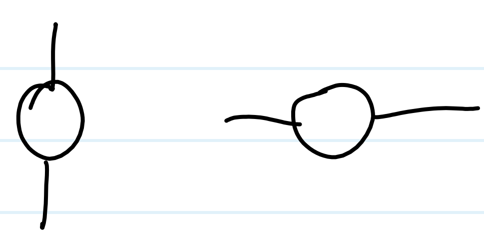
Big trees are scary, but when you come across a source you just have to ask yourself:

1. Independent or Dependent?
2. Voltage or Current?

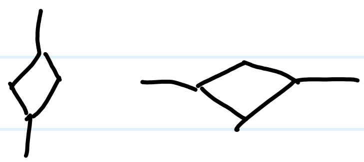
But regardless of the type, a source adds something to a circuit.

Independent or Dependent?

Independent sources are the easy ones, they spit out a value and just continue to spit out the same value forever and ever. Their circuit symbol is:



Dependent sources are more tricky, their output is based on another part of the circuit. They may be either current controlled <u>or</u> voltage controlled by some factor: Out = Constant • Controlling variable. the output changes as it is dependent on other elements in the circuit. Their circuit signal is:



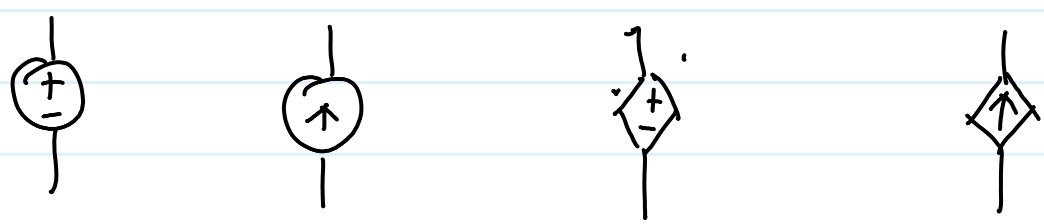
Current or Voltage?

There isn't too much fanciness going on here. Current sources supply a current to the circuit. Voltages supply a voltage.

Current is shown using an arrow in direction of flow "$\rightarrow$"

Voltage is shown using a plus and minus for high and low "+ −"

Wrapping it all up, our 4 types of sources are shown below:



Ind. Voltage   Ind. Current   Dep. Voltage   Dep. Current

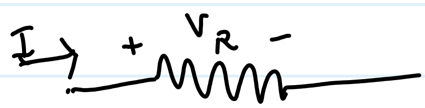
$\downarrow$

$V = \alpha i_\Delta$ would be current controlled

$V = \alpha v_\Delta$ would be voltage controlled

Resistor:

There's much less going on with this guy than their was with the sources. There's only 1 type of resistor in our circuit. It's circuit symbol is:



Capacitors don't do much, they just dissipate power. Their polarity isn't set, it's drawn using the PSC.

But, This is our first real circuit element. Now it's time to introduce our first component equation. This equation is vital to alot of things we do in this class, and it is!

Ohm's Law: $V = IR$

Where:

V = Voltage across resistor, not the voltage at one terminal (V)
I = Current through resistor (A)
R = Resistance value (Ω)

Do you remember earlier when we discussed the definition of a "linear" component? Where the relationship and voltage and current needed to form a linear graph? I gave you a general equation $V = \alpha I$ to summarize it, can you see how ohm's law (and the resistor it describes) is a linear device? R is our constant of porportionality

The last thing to mention about resistors, is that ohms law gives us a few additional ways to calculate the power dissipated by the resistor:

Resistor Power:

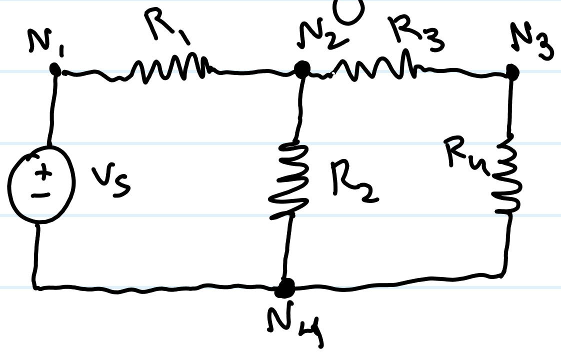
$P = VI$: This is the standard equation for power

$V = IR;\ P = VI \Rightarrow P = (IR)I \Rightarrow P = I^2R$: Power only knowing current

$V = IR \Rightarrow I = \frac{V}{R};\ P = VI \Rightarrow P = V\left(\frac{V}{R}\right) \Rightarrow P = \frac{V^2}{R}$: Power only knowing voltage

Circuit Parts:

For this last portion on the basics of a circuit, lets take a look at the following:



5 Branches:	4 Nodes:	3 Loops:	2 meshes:
B_{12} B_{14}	N_1 N_2	L_{124}	L_{124}
B_{23} B_{24}	N_3 N_4	L_{234}	L_{234}
B_{34}		L_{1234}	

There are a few parts to be able to identify here!

- **Node** - A point in the circuit connecting 2 or more components
- **Branch** - A circuit element or path that connects 2 nodes
- **Essential Node** - A node connecting 3 or more branches
  - N_2 & N_4 are essential nodes
- **Essential Branch** - The path, that may be made of multiple branches, that connects 2 essential nodes without passing through any other essential nodes
  - B_{214}, B_{234}, B_{24} are all essential branches

Loop - A closed path in the circuit

Mesh - A loop in the circuit that does not contain any sub-loops

Reference Node - The node that all other potentials are measured against. For simplicity, it's usually chosen to be the node with a lot of things connected to it.

In our diagram N_4 is most likely to be the reference node

In addition, there's two important types of connections to be aware of and understand

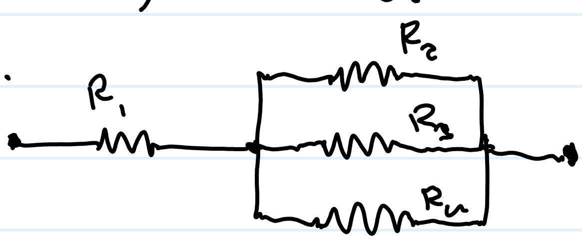
Series - Two components are in series if they share <u>one</u> node, and have no other branches coming from that node.

R_3 & R_4 are in series

R_1 & R_2, R_1 & R_3 are not.

Parallel - Two or more components are said to be in parallel if they share <u>both</u> of their nodes. Other components can (and probably should) branch from these nodes.

R_2 & R_4 are <u>not</u> in parallel, but if we replaced R_3 with a wire they would be.



In the above diagram R_2, R_3, & R_4 are all in parallel.

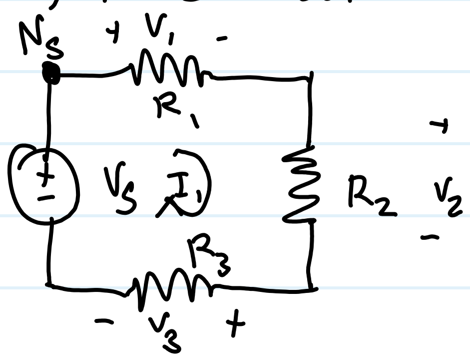
It's important to note, in something like:

there is <u>no</u> series <u>or</u> parallel connections.

Kirchoff's Laws:

In the 1800's Gustav Kirchoff, at the age of 21, published a paper that contained two now incredibly famous equations. He was studying mathematical physics, and thanks to his work, our jobs of circuit analysis are much easier.

The first one we're going to discuss is Kirchoffs Voltage Law (KVL). I'm going to wait to give that a formal definition, because that would be too easy, and would take all the fun out of an intuition. While you wait, take a look at the circuit below:



Since V_S is delivering a voltage to the circuit, we can assure that all of our nodes have some voltage. For our purposes, lets assume $V_S = 5V$, $V_1 = 1V$, $V_2 = 2V$, $V_3 = 3V$. Now lets go around the loop repeatedly, summing the voltages while following the PSC.

Starting from N_s:

- The voltages across resistors will be positive, as they dissipate power / creat voltage drops
- The voltages through the supply will be negative as they deliver power / increase the voltage / potential

Pass 1:

$$N_s = V_1 + V_2 + V_3 - V_5$$
$$= (1) + (2) + (3) - (5)$$
$$= 6 - 5$$
$$N_{s_1} = 1\ V$$

Pass 2: (Carry voltage from last iteration)

$$N_s = N_{s_1} + V_1 + V_2 + V_3 - V_5$$
$$= (1) + (1) + (2) + (3) - (5)$$
$$= 7 - 5$$
$$N_{s2} = 2\ V$$

Pass 3: (Carrying voltage from last iteration)

$$N_s = N_{s2} + V_1 + V_2 + V_3 - V_5$$
$$= (2) + (1) + (2) + (3) - (5)$$
$$= 8 - 5$$
$$N_{s3} = 3\ V$$

So we're running into a bit of a problem here. The fact that the amount of power we're dissipating doesn't match the amount being delivered means that on every iteration we're creating a bigger debt, (remember, positive voltage = Dissipation) until we eventuall create a black hole, consuming not just our circuit, but the entire universe.

This is ... not ideal.

Put more simply, if our inputs, the voltage rises, don't equal our outputs, the voltage drops, we're going to end up in a place of either perpetual motion (Output > Input) or disappearing power (Input > Output), two cases I've been told are not possible under the laws of physics

With this in mind, we arrive at the very simple concept that serves as the basis for both of Kirchoffs laws (and hopefully your checkbook): the inputs must equal the outputs. This more formally is called the Conservation of Energy.

Conservation of Energy - Energy cannot be created or destroyed

If our outputs are greater than our inputs, we're destroying energy. If our inputs are greater than our outputs, we're creating energy.

So, going back to our loop, and readjusting the values, so that $V_S = 5V$, $R_1 = 1V$, $R_2 = 2V$, $R_3 = 2V$, we get the following equation:

$$
\begin{aligned}
N_S &= V_1 + V_2 + V_3 - V_S \\
&= (1) + (2) + (2) - (5) \\
&= 5 - 5 \\
N_S &= 0V
\end{aligned}
$$

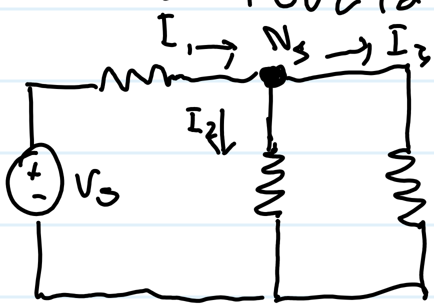
And now, all is right in the world, we can go around infinitely without breaking the universe, and with that, we arrive at the first of Kirchoffs laws

$$KVL: \Sigma V_{loop} = 0$$

Kirchoffs Voltage Law: The sum of the voltage drops (following PSC) around a closed loop equals 0

This law says that when going around a closed loop, the voltages supplied (inputs) have to equal the voltage drops (outputs) to follow the conservation of energy, otherwise you end up in a spot like we saw earlier, creating deficit or supply on every loop.

Congratulations, at this point, you have dissected (and hopefully understood) the more complex of Kirchoff's two laws. We conserved voltage, but now it's time to take a look at the other big element of a circuit, the current. As we talked about in Kirchoffs voltage law, we have to conserve energy, but this time, we're gonna look at a node rather than a loop.



If we look at the currents of N_s, we can form this equation:

$$N_s = -I_1 + I_2 + I_3$$

(Don't forget PSC: Addition is negative, Removal is positive)

If the sum of those 3 values doesn't equal zero, then the node, a wire joint between two components, is either storing or delivering a current to the circuit, which is simply impossible, unless you've built your circuit using some special magic wire that doesn't believe in physics.

So, we again arrive in the same scenario, to obey the conservation of energy our inputs must equal our outputs, and we find the second of Kirchoffs two laws:

KCL: $\sum I_{in} = \sum I_{out}$ or $\sum I_{in} = 0$ (following PSC)

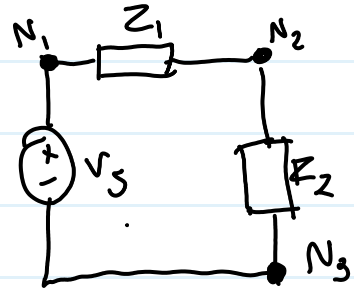
Kirchoffs Current Law! The sum of the currents into a node must equal the sum of the currents out of a node.

And with that, you now know the absolute fundamentals of this course, the things that define everything else. They are the building blocks, so you should become completely comfortable with them, or everything else will be shaky at best.

Unit 1, Section 2: Equivalent Circuits!

Fundamental Combinations:

Before we can really discuss how to combine and transform things, we need to get a grasp on how things like voltages and currents go through a circuit, and how they combine. First, lets take a look at a series circuit and do a bit of inspection:



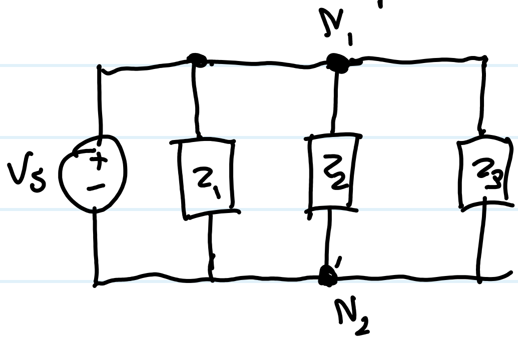
The first thing you might notice about this, is that there's only one path, wich is a requirement for series. If we apply KCL at any (or all) of the nodes, we can see that the current through every component is the same, there's simply nowhere else for the current to go, so if it ever dropped, we'd be breaking the conservation of energy. This is the key element of series connections!

Series Connections:

Same Current

Different Voltages

Now lets look at parallel and do the same process:



It worked for series, lets apply KCL at N_1 and see if it works:

$$\sum I_{in} = 0$$
$$-I_S + I_{Z_1} + I_{Z_2} + I_{Z_3} = 0 \quad : \text{Following PSC}$$
$$I_S = I_{Z_1} + I_{Z_2} + I_{Z_3}$$

Okay, well, that didn't work, there's no rule saying those 3 currents have to be the same. Lets instead try KVL for the loops containing the voltage supply and 1 source:

$$\sum V_{Loop} = 0 \qquad \sum V_{Loop} = 0 \qquad \sum V_{Loop} = 0$$
$$-V_S + V_{Z_1} = 0 \qquad -V_S + V_{Z_2} = 0 \qquad -V_S + V_{Z_3} = 0$$
$$V_S = V_{Z_1} \qquad V_S = V_{Z_2} \qquad V_S = V_{Z_3}$$

Aha! Now we're getting somewhere, KCL didn't yield anything, but using KVL, we can see that all of the components in parallel must have the same voltage!

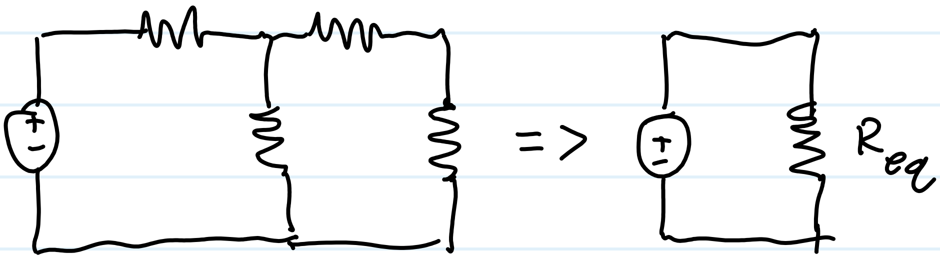
Parallel Connections:

Same Voltage

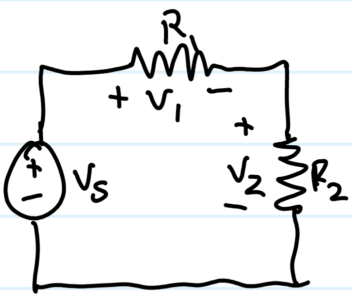
Different Currents

Equivalent Resistances:

In our circuits, as long as we follow rules, we can come up with a general way to combine resistors to represent a network as one large resistance R_{eq}!



Knowing what we know, lets find a general formula for how resistors combine, starting with series:



In series, currents are the same, use KVL

KVL:

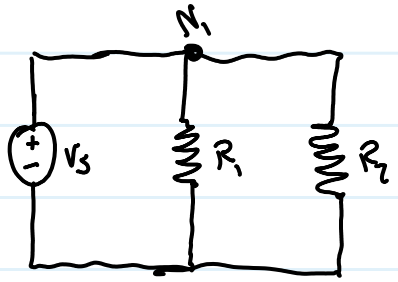
$$-V_s + V_1 + V_2 = 0$$
$$V_s = V_1 + V_2$$
$$V_s = (IR_1) + (IR_2) \quad \text{: Ohm's law } (V = IR)$$
$$V_s = I(R_1 + R_2) \quad \text{: Extract } I$$
$$\frac{V_s}{I} = R_1 + R_2$$
$$R_{eq} = R_1 + R_2 \quad \text{: Ohm's Law } \left(\frac{V}{I} = R\right)$$

From this we can kinda see the following

Series Equivalent Resistance:

$$R_{eq} = \Sigma R_n$$

Now lets repeat this process for parallel connections!



In series, voltages are the same, use KCL

N_1:

$$-I_s + I_1 + I_2 = 0$$

$$I_s = I_1 + I_2$$

$$\left(\frac{V_s}{R_{eq}}\right) = \left(\frac{V_s}{R_1}\right) + \left(\frac{V_s}{R_2}\right)$$: Ohm's law ($\frac{V}{R} = I$)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$: Cancel V_s

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

Parallel Equivalent Resistances!

$$R_{eq} = \left(\sum \frac{1}{R_n}\right)^{-1}$$

Special Case: 2 Parallel Resistors!

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

$$= \left(\frac{R_2}{R_1R_2} + \frac{R_1}{R_1R_2}\right)^{-1}$$

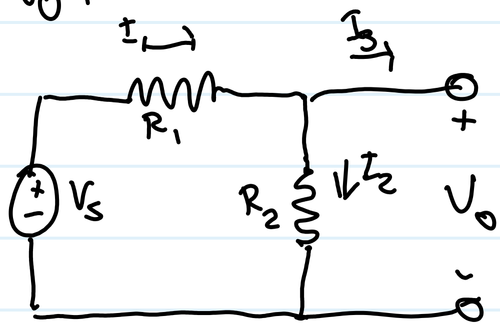
$$= \left(\frac{R_1 + R_2}{R_1R_2}\right)^{-1}$$

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$$

You may see this notation: $R_1 // R_2$. This means that R_1 is in parallel with R_2

Voltage and Current Dividers!

Lets take a look at the following circuit, and find an expression for V_0:



Things to note:

V_0 is an open, so $I_3 = 0$

W/ $I_3 = 0$, R_1 & R_2 are in series.

$I = I_1 = I_2$

V_0 is the voltage across R_2

A really simple answer would be that $V_0 = I R_2$, but lets see if we can get everything in terms of given constants!

KVL:

$$-V_S + V_1 + V_2 = 0$$

$$V_S = V_1 + V_2$$

$$V_S = IR_1 + IR_2 \quad : \text{Ohm's law } (V = IR)$$

$$V_S = I(R_1 + R_2)$$

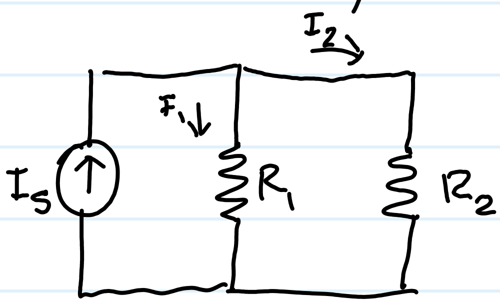
$$I = \frac{V_S}{R_1 + R_2}$$

Now that we have I, if we sub that back in to our simple answer we get

Voltage Divider Formula: $V_0 = V_S \dfrac{R_2}{R_1 + R_2}$

We've found a way to get the output voltage completely in terms of the input and a resistor ratio! This forms a common circuit called a voltage divider, and is super useful for turning a high voltage into a lower one by choosing different component values.

Well, can we try it again, but with current? Our resistors can't be in series anymore, but if we put them in parallel? Take a look at the circuit below, and try to find an expression for I_2



KCL:

$$-I_S + I_1 + I_2 = 0 \quad : \text{KCL}$$

$$I_S = I_1 + I_2$$

$$I_S = \left(\frac{V}{R_1}\right) + \left(\frac{V}{R_2}\right) \quad : \text{Ohms law } \left(I = \frac{V}{R}\right);\ \text{Parallel} = \text{Same Voltage}$$

$$I_S = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$V = I_S\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

$$V = I_S\left(R_1 // R_2\right) \quad : R_p = \left(\Sigma R_n\right)^{-1}$$

$$V = I_S\left(\frac{R_1 R_2}{R_1 + R_2}\right) \quad : \text{Special Case of 2 Resistors}$$

With the equation for voltage, we can sub that into the ohm's law of any resistor to find it's current:

$$I_1 = \frac{V}{R_1} = \frac{1}{R_1}\left(I_S\left(\frac{R_1 R_2}{R_1 + R_2}\right)\right) = I_S \frac{R_2}{R_1 + R_2}$$

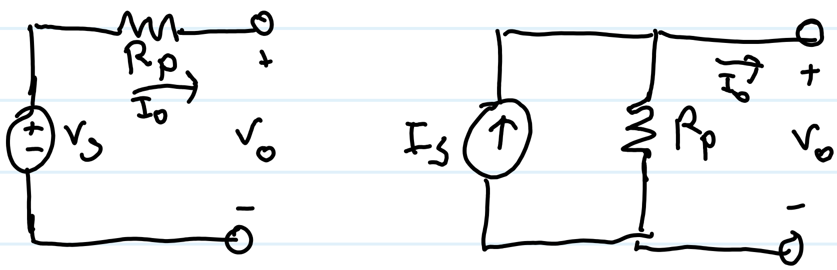
$$I_2 = \frac{V}{R_2} = \frac{1}{R_2}\left(I_S\left(\frac{R_1 R_2}{R_1 + R_2}\right)\right) = I_S \frac{R_1}{R_1 + R_2}$$

Current Division:

$$I_n = \frac{I_S}{R_n} \cdot \left(R_1 // R_2 // \ldots // R_n\right) = I_S \frac{R_{eq}}{R_n}$$

Source Transformations!

After toying with ohms law and Kirchoff's law, you can kinda get the sense that, to some degree, the circuits below are equivalent:



And if they are equivalent, there should be some way to convert between them, so lets try and find a way to do just that. Looking at the two circuits, lets try and find an equation for Vo and I_o.

I_o is the current to V_o, but in both circuits, V_o is an open, so it will be equal to zero in both circuits. Vs, however, is not. In the circuit with the voltage source, since there is no current, the voltage of the resistor is also zero, meaning $V_o = V_S$. In the circuit with the current source, Vo is the voltage across the resistor, which by ohm's law is $V_o = I_S R_P$. But since V_o also equals V_s, we get

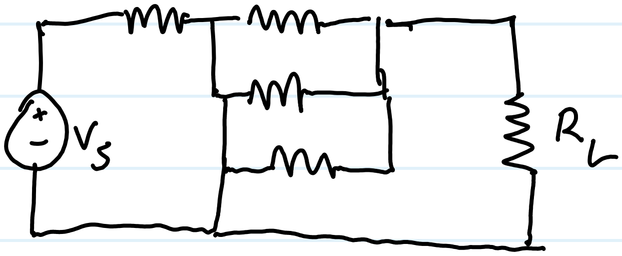
Source Transform: $V_S = I_S R_p$

But now you may be asking yourself, "well couldn't R_p in the voltage circuit be any value?" No! That's fine here, but you have to remember, this transform is just a part of the circuit. If R_p isn't the same between both, you're going to affect the rest of your circuit. R_p must be the same in both circuits

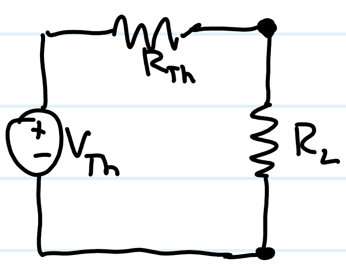
Thevinin & Norton Equivalent:

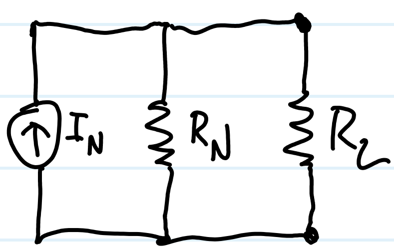
Pre-Req: Superposition, Mesh Current, Node Voltage

So lets say, hypothetically, you've got a big resistor network connected to a load, like below!



This is large. It's complex. and quite frankly, it's annoying to draw. Luckily, there was a very nice engineer that came up with a clever way to solve this problem, changing this into one of the two forms!





Process:

1. Zero all sources.
2. Find equivalent resistance from terminals
3. Add sources back in
4. If thevenin: Voltage across terminals, if Norton: S.C. Current
5. Make new circuit

Unit 1, Section 3: Analysis Methods

Basics of Circuit Analysis:

Suppose I were to give you a circuit. Any circuit, and say "solve it." What does that mean? Well, just like in geometry, it means we want to find every unknown of the circuit, which is all of the currents and voltages at every node and through every component. How would you achieve this?

Well, you've been given the fundamentals already, pretty much every technique is based on Ohm's law, KVL, and KCL, it's just about applying them in a way that doesn't create a bunch of extra work. So, to solve this in the easiest way possible, we have to keep in mind a concept that we learned in algebra: For N unknowns, you can create N linearly independent equations.

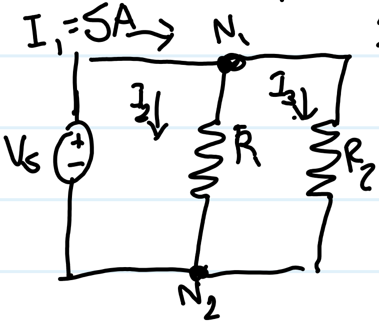
Independent Equations: A set of equations that cannot be written as scalar (real #) multiples of each other

$x+y=1$ & $x+2y=2$ are linearly independent

$x+y=1$ & $2x+2y=2$ are linearly dependent.

Put another way, given n linearly independent equations you can only find n unknowns, so if 2 or more of your equations are dependent on each other, you can't find all of the unknowns you need.

Where dependencies happen and what do they look like? Well, here's a perfect example below; I'm gonna try and solve for I_2 and I_3 w/only KCL



2 Nodes

N₁: $-I_1 + I_2 + I_3 = 0$

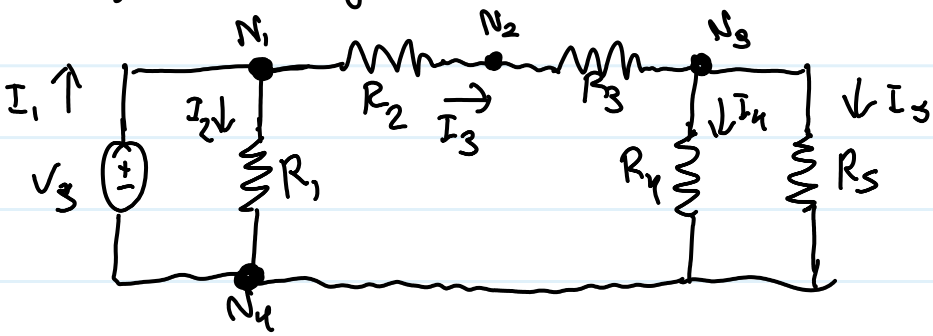
N₂: $I_1 - I_2 - I_3 = 0$

We have our two equations, but if you try to solve this system, you'll very quickly notice, they are actually the same: $I_2 + I_3 = 5$, and you need one of the two values to solve for the other. This is an important note

KCL Restriction: For N unknowns, we have N-1 independent KCL Equations

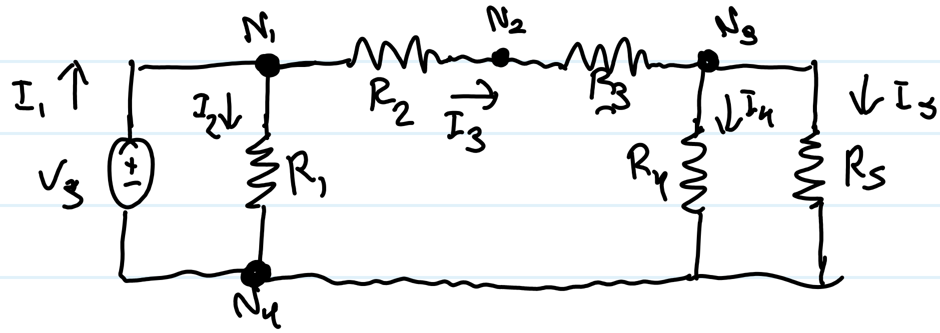
Node-Voltage Method:

Both of the next two approaches we're gonna learn about are going to seem a little counterintuitive at first, but if you let your mind play with the ideas, they will eventually make sense. This first one is going to use KCL with ohm's law to find voltages, and is called node voltage analysis. Take that last sentence and see if you can come up with a guess for how this might work, and then take a look below, does my labelling of nodes make sense?



Before doing anything real, we're going to want to assign a reference node, because not everything can be energized, otherwise you're not going to be conserving energy. If you want to see this in action, try to find an equation for the loop containing only V_s and R_1 that leaves some voltage at both nodes N_1 and N_4. You can't while conserving energy, so step one is always to assign a reference node, which since it's zero, we usually choose to be the most complex node, as it just removes it, and in this case it would be N_4. By doing this, we also immediately solve for one of our unknowns, meaning in pretty much every case, the N-1 constraint on KCL isn't an issue any more.

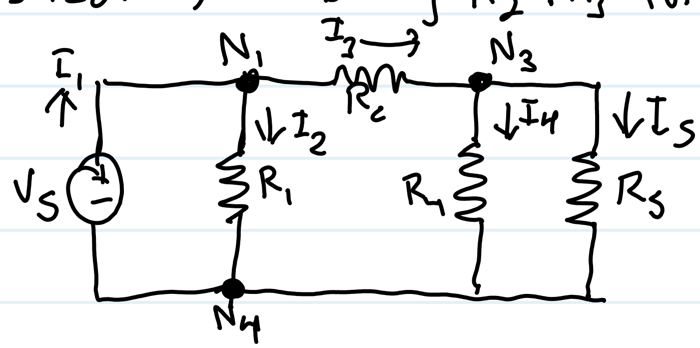
OKay, so now we have this, lets apply KCL at the nodes and see what we find



where $N_4 = 0$ V

$\underline{N_1}$: $-I_1 + I_2 + I_3 = 0$ $\underline{N_2}$: $-I_3 + I_3 = 0$ $\underline{N_3}$: $-I_3 + I_4 + I_5 = 0$

The first big thing to note here is that the N_2 equation is useless (the nicer term is "trivial") all it tells us is that $I_3 = I_3$, which... we know. And this brings us to our second point on Node-Voltage: You apply KCL at essential nodes, which are those with 3 or more connections to them, otherwise we're always going to see equations like N_2. So now lets redraw, combining R_2 & R_3 for simplicity!



$\underline{N_1}$: $I_1 = I_2 + I_3$

$\underline{N_3}$: $I_3 = I_4 + I_5$

Now this is where things get weird, we're going to use ohms law to get the voltages of every node. I'm going to write down the conversion of each current, make sure it makes sense before moving on, this is the hardest but most important step of node voltage analysis. Don't forget: Ohms law! $V = IR$ and voltage is the voltage difference between a components terminals NOT the voltage at one terminal.

I_1: We don't know, it's the current through the voltage source, but there's no resistor so we can't use Ohms law

I_2: $\frac{V_1}{R_1}$, I_3: $\frac{V_1 - V_3}{R_C}$, I_4: $\frac{V_3}{R_4}$, I_5: $\frac{V_3}{R_5}$

With that done we can rewrite our equatons as:

$$I_1 = I_2 + I_3 \qquad I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_3}{R_C}$$
$$I_3 = I_4 + I_5 \implies \frac{V_1 - V_3}{R_C} = \frac{V_3}{R_4} + \frac{V_3}{R_5}$$

So the top equation isnt super useful yet, we can't get I_1 in terms of voltages yet. But what we can solve for (by inspection) is V_1, which has to equal V_S considering that there's nowhere for the voltage to go between the source and the node. Now it may technically be an unkown, but we'd be stuck if we don't have any more data, so we're going to assume that V_S is given. Rewriting again:

$$I_1 = \frac{V_S}{R_1} + \frac{V_S - V_3}{R_C} \quad ; \quad \frac{V_S - V_3}{R_C} = \frac{V_3}{R_4} + \frac{V_3}{R_5}$$

So, Next we need to solve for V_3 to find the rest of our unkowns. We can use our second equation with a bit of algebra to achieve this:

$$\frac{V_S - V_3}{R_C} = \frac{V_3}{R_4} + \frac{V_3}{R_5} \implies \frac{V_S}{R_C} - \frac{V_3}{R_C} = \frac{V_3}{R_4} + \frac{V_3}{R_5} \implies \frac{V_S}{R_C} = \frac{V_3}{R_4} + \frac{V_3}{R_5} + \frac{V_3}{R_C}$$

$$\implies \frac{V_S}{R_C} = V_3\left(\frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_C}\right) \implies V_3 = \frac{V_S}{R_C}\left(\frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_C}\right)^{-1}$$

And with that, we have found V_3 in terms of known variables.

From these, we could solve for any quantity in the circuit we want as every current and voltage can be expressed in terms of quantities we already know. For simplicity, here's a summary of the process:

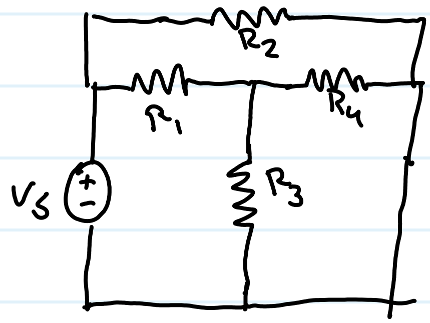
Node-Voltage Analysis:

1. Identify and label all essential nodes and circuit currents
2. Assign a reference node to have a voltage of 0
3. Apply KCL at each node
4. Use Ohm's law to rewrite currents in terms of the node voltages
5. Solve easy voltages by checking for any nodes with connected supplies
6. Make algebra your best friend and solve for the rest of your unknowns

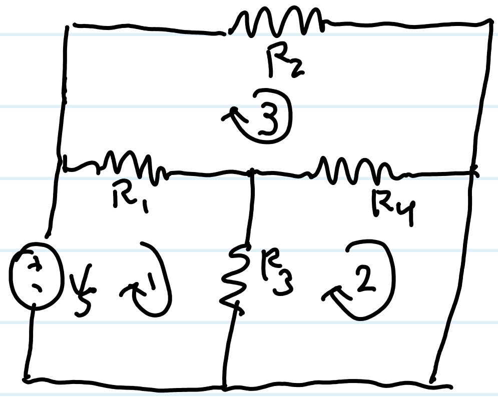
Node voltage does work for solving the whole circuit, but the key concept of rewriting a current in terms of voltages is something we'll do much more frequently, as a little mini-analysis, so make sure you're comfortable with that.

Mesh Current Analysis:

Now it's time for the second of our two "weird" approaches: mesh current analysis. Like the last one, this uses Ohm's law to make an interesting conversion, but this time it's using the voltages produced by KVL to find currents. As the name suggests, we're going to use Meshes, as they are guaranteed to be linearly independent. So, to get this started, consider the circuit below:

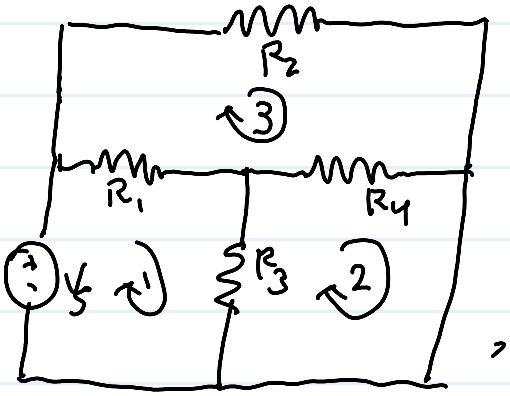


Now, we're going to identify the meshes in the circuit, and decide the current direction in them, try it and see if you get something that looks like:



Do note that the direction is arbitrary, you can choose whatever direction you'd like as long as you're consistent

Next, we're going to apply KVL around each of the meshes, this should be relatively easy...



1: $-V_S + V_1 + V_3 = 0$
2: $V_3 + V_4 = 0$
3: $V_2 + V_1 + V_4 = 0$

Except surprise, it isn't. If you do like I did, you'll notice that my signs on resistors 1, 3, & 4 aren't consistent. I enter both sides of them and treat it like a drop both ways, which can't be possible, entering one side has to be a drop, and the other a raise. Thats because, when I was writing them, I reassigned the polarity on every mesh travel. The alternative would be assigning a more permanent polarity, but then you have to track your signs. After I do the Ohms law swap, I'll show that the two methods are equivalent. Just like with node voltage, this step is the trickiest but most important, So I will list every transform and make sure it makes sense. It relies on the fact that the current through the resistor is equal to the total sum of the currents passing through it!

1:

V_S: Again, this is a source, can't use Ohm's law
V_1: $(I_1 - I_3)R_1$
V_3: $(I_1 - I_2)R_3$

②:

$V_3 : (I_2 - I_1) R_3$

$V_4 : (I_2 - I_3) R_4$

③:

$V_2 : I_3 R_2$

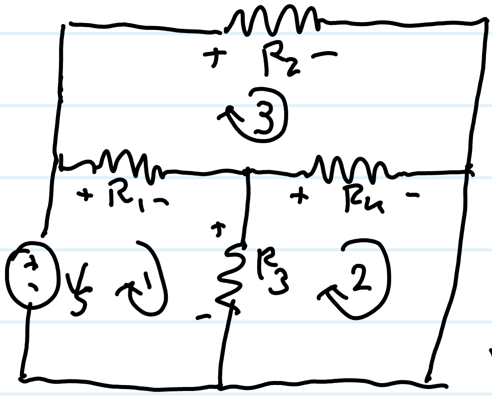
$V_4 : (I_3 - I_2) R_4$

$V_1 : (I_3 - I_1) R_1$

Putting it all together, lets sub the back into our original system:

$$\begin{aligned} V_S &= V_1 + V_3 \\ 0 &= V_3 + V_4 \\ 0 &= V_1 + V_2 + V_4 \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_3 &= (I_1 - I_3) R_1 + (I_1 - I_2) R_3 \\ 0 &= (I_2 - I_1) R_3 + (I_2 - I_3) R_4 \\ 0 &= I_3 R_2 + (I_3 - I_2) R_4 + (I_3 - I_1) R_1 \end{aligned}$$

With that done, lets take ~~a~~ second to do this with the fixed polarities



①: $-V_S + V_1 + V_3 = 0 \Rightarrow V_S = V_1 + V_3$

②: $-V_3 + V_4 = 0$

③: $V_2 - V_4 - V_1 = 0$

$V_1 = (I_1 - I_3) R_1$, $V_2 = I_3 R_2$, $V_3 = (I_1 - I_2) R_3$, $V_4 = (I_2 - I_3) R_4$

$$\begin{aligned} V_S &= V_1 + V_3 \\ 0 &= -V_3 + V_4 \\ 0 &= -V_1 + V_2 - V_4 \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_S &= (I_1 - I_3) R_1 + (I_1 - I_2) R_3 \\ 0 &= -(I_1 - I_2) R_3 + (I_2 - I_3) R_4 \\ 0 &= -(I_1 - I_3) R_1 + I_3 R_2 - (I_2 - I_3) R_4 \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_S &= (I_1 - I_3) R_1 + (I_1 - I_2) R_3 \\ 0 &= (I_2 - I_1) R_3 + (I_2 - I_3) R_4 \\ 0 &= (I_3 - I_1) R_1 + I_3 R_2 + (I_3 - I_2) R_4 \end{aligned}$$

So, by comparing our results we can see that we end up at the same system. But which approach is better? Well, the first gets tricky if you have to manually write down all of the ohm's laws, as you have to be careful to make the right, path dependent, substitutions or everything is going to be wonky. But! If you can do the ohm's law equations in your head, then you can just mark everything as a voltage drop, and your signs will be handled for you. If you're not comfortable doing the substitution without writing it down, I recommend assigning polarities, because then there's only one subsitution for each symbol, you will just have to distribute your negatives. But anyway, lets keep going, we've completely represented our circuit so for simplicity, I'm going to work with just the equations. In the end, if we solve our system, we could solve for any unknown in the circuit, but with 3 unknowns, I'm going to get it into matrix form:

$$V_S = (I_1 - I_3)R_1 + (I_1 - I_2)R_3$$
$$0 = (I_2 - I_1)R_3 + (I_2 - I_3)R_4$$
$$0 = I_3 R_2 + (I_3 - I_2)R_4 + (I_3 - I_1)R_1$$

$$\Rightarrow \begin{aligned} V_S &= I_1 R_1 - I_3 R_1 + I_1 R_3 - I_2 R_3 \\ 0 &= I_2 R_3 - I_1 R_3 + I_2 R_4 - I_3 R_4 \\ 0 &= I_3 R_2 + I_3 R_4 - I_2 R_4 + I_3 R_1 - I_1 R_1 \end{aligned}$$

$$\Rightarrow \begin{aligned} V_S &= (R_1 + R_3) I_1 + (-R_3) I_2 + (-R_1) I_3 \\ 0 &= (-R_3) I_1 + (R_3 + R_4) I_2 + (-R_4) I_3 \\ 0 &= (-R_1) I_1 + (-R_4) I_2 + (R_1 + R_2 + R_4) I_3 \end{aligned}$$

$$\Rightarrow \begin{bmatrix} R_1 + R_3 & -R_3 & -R_1 \\ -R_3 & R_3 + R_4 & -R_4 \\ -R_1 & -R_4 & R_1 + R_2 + R_4 \end{bmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} V_S \\ 0 \\ 0 \end{pmatrix}$$

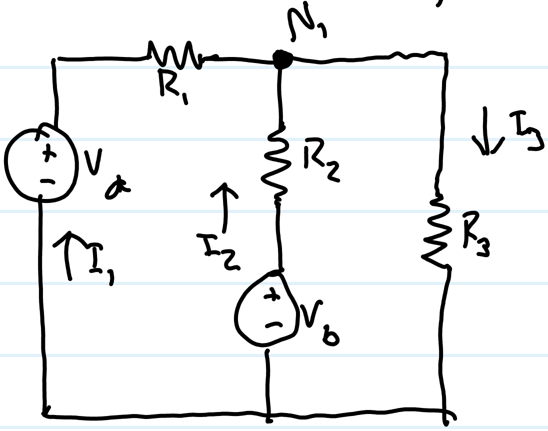
Mesh Current Summary:

1. Identify Meshes and assign current direction
2. Do KVL around Mesh in direction of current
3. Represent KVL equations in terms of component Currents
4. Solve system for any needed quantity or get it into matrix form

Super Position:

This next method is less of an approach, and more just a fact of our circuits, although it can be incredibly useful. Remember earlier when we said all of our components are linear? Meaning if you plotted the relationship between their voltage and current it would be a straight line? Well that's vital here. If your circuits are linear, then you can use this concept.

Here's why, thinking back to what we learned about in Node voltage and mesh current, you can kind of see that the properties of a component can kind of be expressed as a sum of other values. To reinforce this idea, lets take a look at this circuit and do KCL at N_1

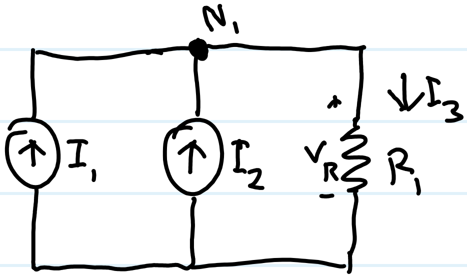


N_1:

$-I_1 - I_2 + I_3 = 0$

$I_3 = I_1 + I_2$

So from this idea we could, if we were careful with it, remake the circuit into something looking like this



N_1:

$$-I_1 - I_2 + I_3 = 0$$
$$I_3 = I_1 + I_2$$

Now, I want to try something and see what happens. Lets say $I_1 = 5\,A$, $I_2 = 2\,A$, and $R_1 = 5\,\Omega$, and find the voltage across the resistor in two ways. The first is the standard way:

$$I_3 = I_1 + I_2 \Rightarrow \frac{V_R}{R} = (5) + (2) \Rightarrow V_R = 7R \Rightarrow V_R = 7(5) = 35\,V$$

The second, we're going to actually analyze it twice, taking turns pretending one of the sources is zero and then adding the two voltages together:

$$I_3 = I_1 + \cancel{I_2} \Rightarrow \frac{V_R}{R} = (5) + (0) \Rightarrow V_{R\alpha} = 5R = 5(5) = 25\,V$$
$$I_3 = \cancel{I_1} + I_2 \Rightarrow \frac{V_R}{R} = (0) + (2) = V_{R\beta} = 2R = 2(5) = 10\,V$$
$$V_R = V_{R\alpha} + V_{R\beta} = (25) + (10) = 35$$

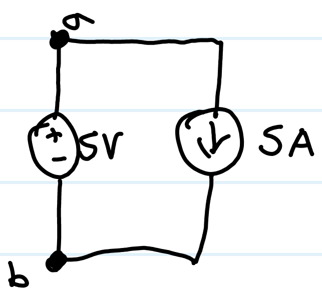
Huh, that's interesting, we could make each of the sources zero and sum the results, and get the same answer as if we had done it all connected. This is the concept of superposition, and here's why it works:

$$V_R = IR \Rightarrow V_R = (I_1 + I_2 + \ldots + I_n)R \Rightarrow V_R = \overbrace{I_1 R}^{(V=IR)} + I_2 R + \ldots + I_n R$$
$$= V_R = V_1 + V_2 + \ldots + V_n$$

So, for any linear circuit, you can zero out all but one source, perform your analysis, and then repeat this for every other source, summing your results in the end. Kind of neat right?

But what does it mean to zero a source? Well, it means we want to reconfigure the circuit as if that source didn't do anything. Let's try and figure that out, starting with the voltage source.

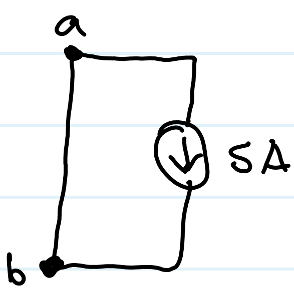
The voltage source raises the potential between two nodes, without affecting the current. So if we had:



$\Delta V_{ab} = 5V$

$I_{ab} = 5A$

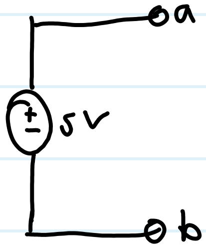
In order to charge the voltage without affecting the current, we would need to replace the voltage source with a wire



$\Delta V_{ab} = 0V$

$I_{ab} = 5A$

Similarly, the current source raises the current without affecting the potential, and to achieve that we'd need to use an open circuit!



$\Delta V_{ab} = 5\,V$

$I_{ab} = 0A$

Zeroing Sources Summary:

Zeroed Voltages are Wires

Zeroed Currents are Opens